(D) Evaluate

$$\int_0^{2\pi} \frac{d\theta}{1+a\cos\alpha}, a^2<1. \qquad 6$$

UNIT—V

5. (A) Prove that a finite point set has no limit point. 1½

(B) If X is a metric space and $E \subset X$, then prove that $\overline{E}$ = closure of E = $E \cup E'$ is closed where $E' \equiv$ the set of all limit points of E in X. 1½

(C) Prove that if F is closed and K is compact, then $F \cap K$ is compact. 1½

(D) If $\{K_n\}$ is a sequence of non-empty compact sets such that $K_n \supset K_{n+1}$ (n = 1, 2, 3, …), then prove that $\bigcap_1^{\infty} K_n$ is not empty. 1½

(E) Let R be a ring. Prove that if $a, b \in R$, then $(a+b)^2 = a^2 + ab + ba + b^2$. 1½

(F) If R is a ring and $a \in R$, let $r(a) = \{x \in R/ax = 0\}$. Prove that r(a) is a right-ideal of R. 1½

(G) Evaluate $\int_0^{1+i} z\,dz$ along the line z = 0 to z = 1 + i. 1½

(H) If $f(z) = \frac{1}{(z-5)^3 (z-4)^2}$ then find the poles with order. 1½

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MATHEMATICS

(M_{10}-Metric Space, Complex Integration and Algebra)

Paper—II

Time—Three Hours] [Maximum Marks—60

N.B. :— (1) Solve all the **FIVE** questions.

(2) All questions carry equal marks.

(3) Solve each question in full or its alternative in full.

UNIT—I

1. (A) Prove that every infinite subset of a countable set A is countable. 6

(B) Let X be an infinite set. For $p \in X$ and $q \in x$, defrine

$$d(p, q) = \begin{cases} 1 & (\text{if } p \neq q) \\ 0 & (\text{in } p = q) \end{cases}.$$

Prove that this d is a metric. 6

OR

(C) Let $\{E_\alpha\}$ be a (finite or infinite) collection of sets E_α. Then prove that

$$\left(\bigcup_\alpha E_\alpha\right)^c = \bigcap_\alpha \left(E_\alpha^c\right)$$ 6

(D) For any collection $\{G_\alpha\}$ of open sets, prove that $\bigcup_\alpha G_\alpha$ is open. Hence, for any collection $\{F_\alpha\}$ of closed sets, prove that $\bigcap_\alpha F_\alpha$ is closed. 6

UNIT—II

2. (A) Prove that closed subsets of compact sets are compact. 6

(B) If $\{I_n\}$ is a sequence of intervals in R^1, such that $I_n \supset I_{nH}$ (n = 1, 2, 3,…), then prove that $\bigcap_1^\infty I_n$ is not empty. 6

OR

(C) Let Y be a subspace of a complete metric space X. then prove that Y is complete if and only if Y is closed. 6

(D) Let a subset E of the real line R^1 be connected and $x, y \in E$ such that $x < z < y$, then prove that $z \in E$. 6

UNIT—III

3. (A) Prove that a finite integral domain is a field. 6

(B) If ϕ is a homomorphism of a ring R into a ring R' with kernel I (ϕ), then prove that

(a) I (ϕ) is a subgroup of R under addition and

(b) if $a \in I(\phi)$ and $r \in R$ then both ar and ra are in I (ϕ). 6

OR

(C) Prove that the homomorphism ϕ of R into R' is an isomorphism if and only if I (ϕ) = kernel of ϕ = {0}. 6

(D) If $\cup$ is an ideal of R, let $r(\cup) = \{ x \in R / xu = 0$ for all $u \in \cup\}$ Prove that r ($\cup$) is an ideal of R 6

UNIT—IV

4. (A) If f(z) is analytic within and on a closed contour $\subset$, and if a is any point within $\subset$, then prove that $f(a) = \frac{1}{2\pi i}\int_c \frac{f(z)\,dz}{(z-a)}$. 6

(B) Find the value of the integral $\int_0^{1+i} (x - y + ix^2)\,dz$ along the straight line from z = 0 to z = 1 + i. 6

OR

(C) Evaluate the residues of $\frac{z^2}{(z-1)(z-2)(z-3)}$ at 1, 2, 3, and infinity and show that their sum is zero. 6

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